بسم الله الرحمن الرحيم

Benha University
Shoubra Faculty of Engineering

Mechanical Department
1st year Mech.

17.03.2019 – Week 6

Torsion test

Prof. Farida Sayed Ahmed
Dr. Mahmoud Khedr

Outline

- Types of stresses.
- Single, Double & Punching shear stresses.
- Torsion test.
- Stress distribution & mode of failure.
- Watching a torsion testing practice.
- The mechanical properties after torsion test.

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Types of stresses

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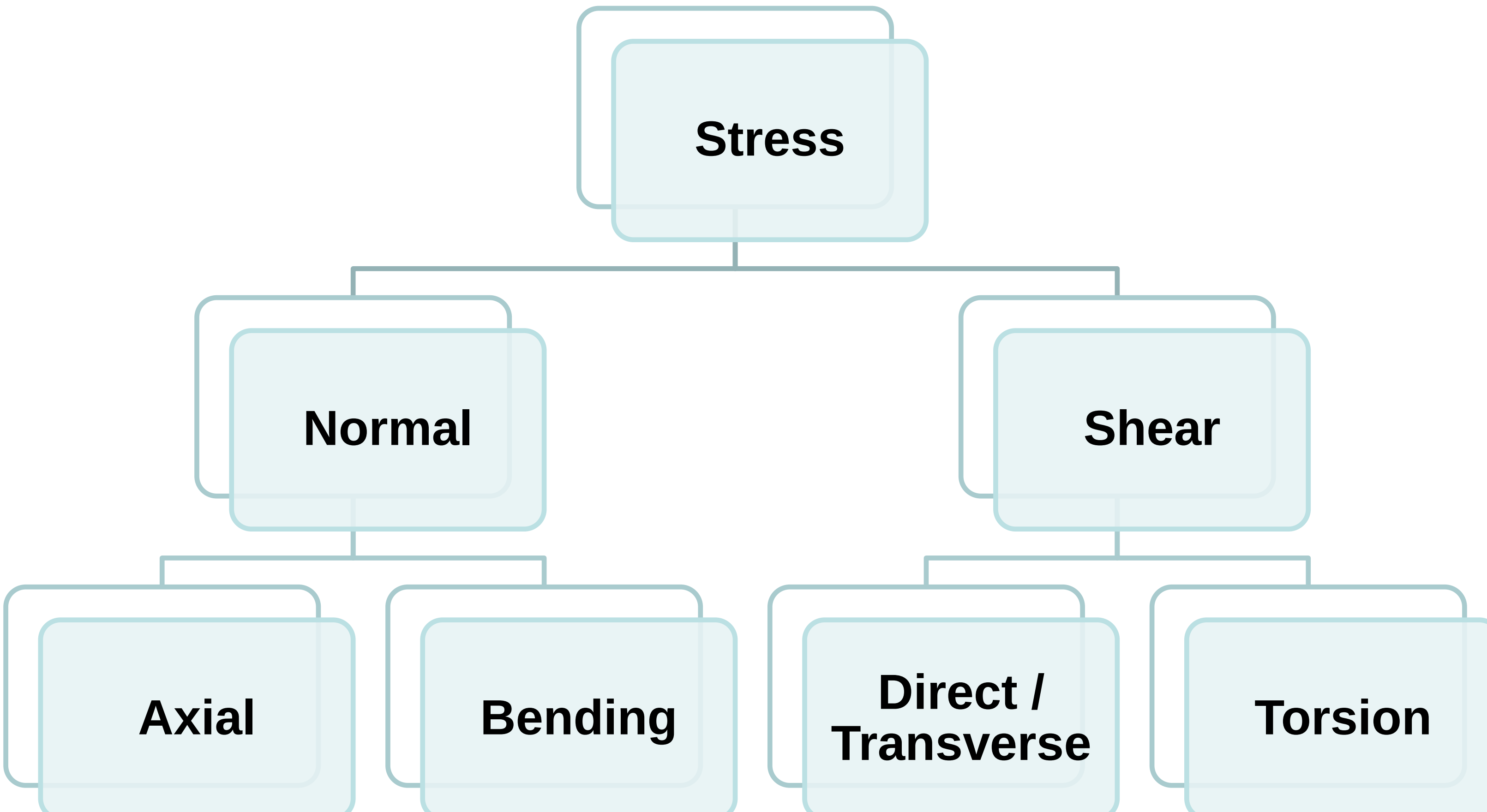
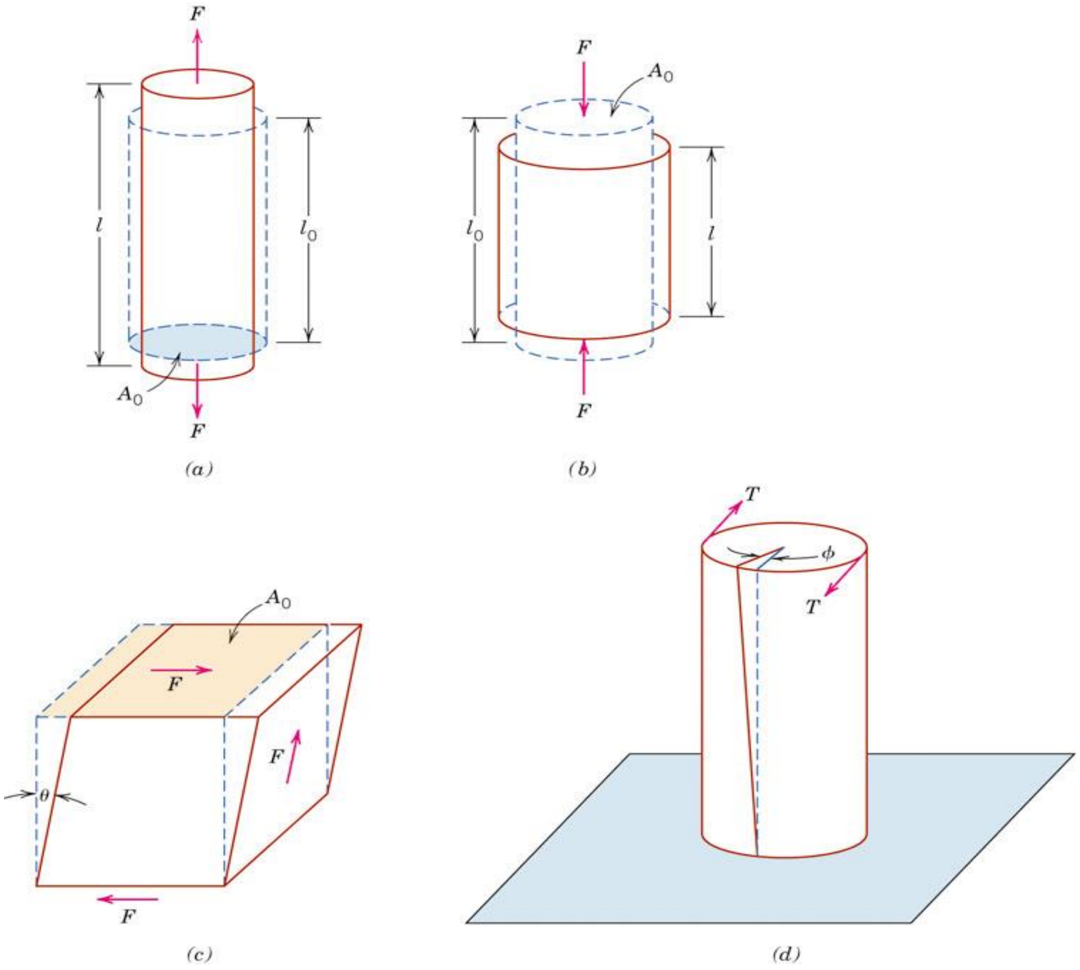
Stresses types



Loading types



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Single, Double & Punching shear

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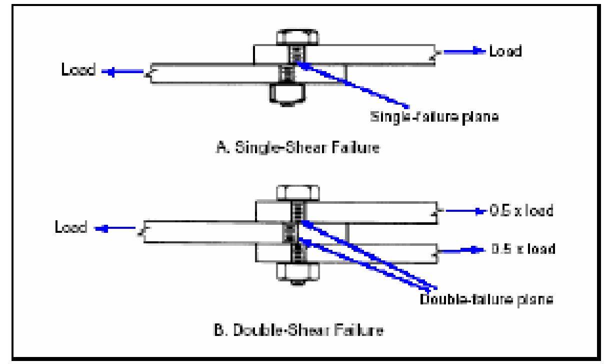
Direct Shear stress

Types of Direct Shear

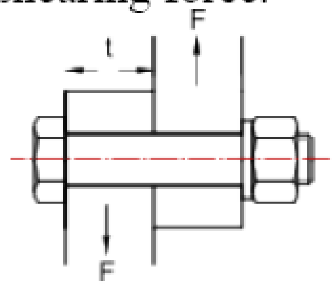
Direct Shear is called single shear if one critical single cross section is subjected to shearing force.

Direct Shear is called double shear if two critical cross sections are subjected to shearing force.



Single Shear

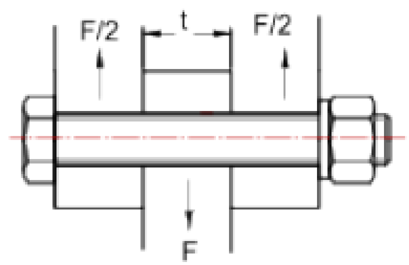
Shear Stress $= 4 \, . \, F / \pi . \, d^2$



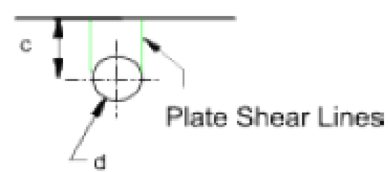
Single Shear

Double Shear

Shear Stress $= 2 \, . \, F / \pi . \, d^2$



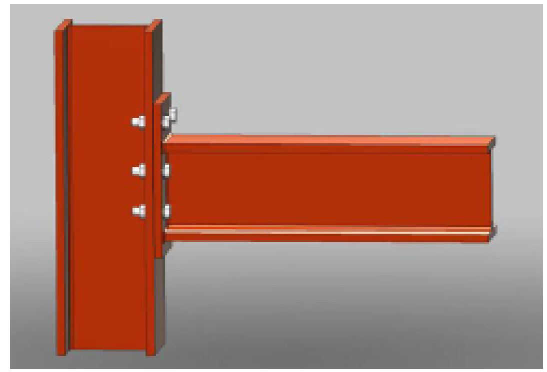
Double Shear



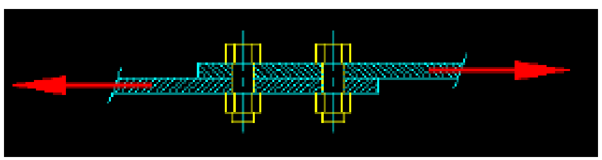
Direct Shear stress

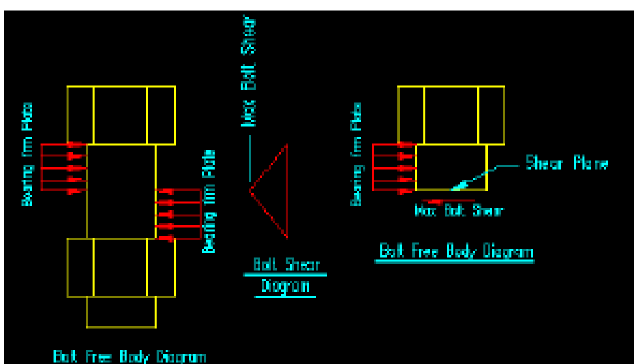
Bolted Connections

Bolted connections are made with nuts and bolts. They can are either be single or double shear connections.



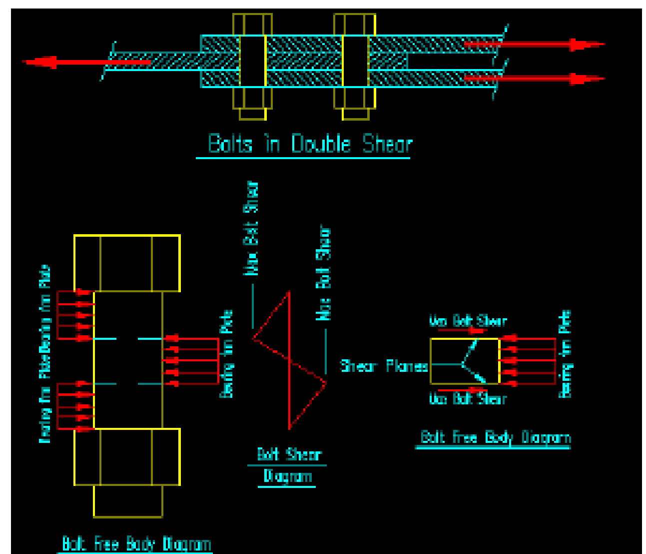
Single shear





Shear Stress = 4 . F / π. d²

Double shear



Shear Stress = 2 . F / π. d²

Direct Shear stress

Mode of Failure for Direct Shear

Direct shear tests are carried out on metals used in making nuts and bolts where the bolts are going to be subjected to direct shear during the normal use.

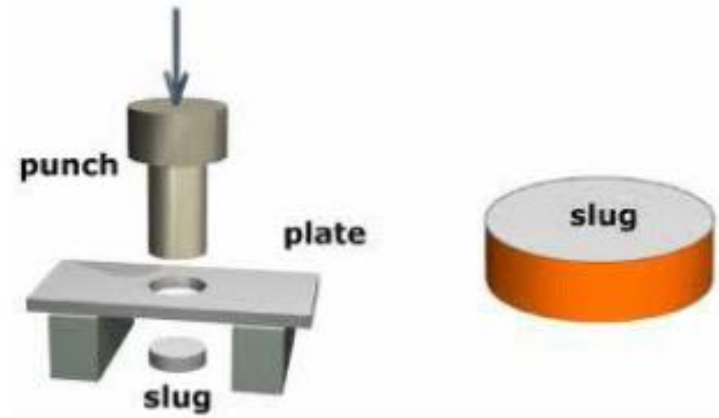
<u>Failure mode consists of two portions:</u>

The first portion has a ***<u>smooth surface</u>*** due to the sliding of planes due to the shearing force.

The second portion is rough due to the ***<u>sudden failure</u>*** of the cross section under the action of the concentrated stresses.

Punching Shear

Punching Shear occurs when a hole is punched under static load in a thin plate producing a hole of diameter Φ and a metal slug.

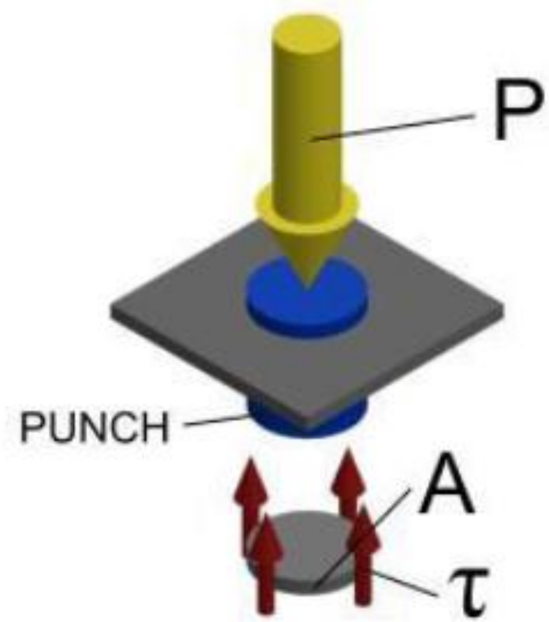


Punching shear stress is calculated as

$$\tau = P / (\pi . \Phi . t)$$

where;
τ = punching shear stress
Φ = hole diameter
t = plate thickness

Punching shear is used in the steel structures industry for plates thinner than 12mm.



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Torsion test

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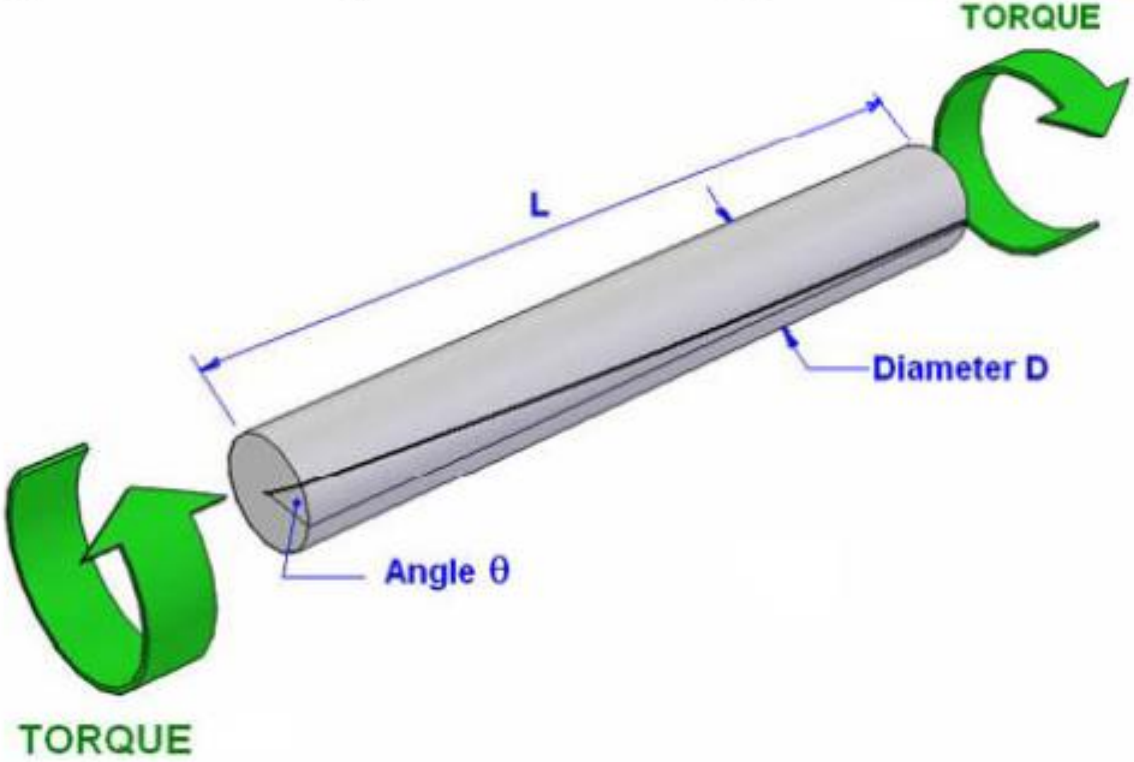
Introduction

Torsion is an important type of loading that can produce pure shear stresses in engineering applications. Under only torsion moment, pure shear stresses are produced while the stiffness under shear stress is called the Modulus of rigidity G.(MPa) It is also called Shear Modulus.

Torsion may produce a sliding state due to the application of ***twisting moment (Torque)***

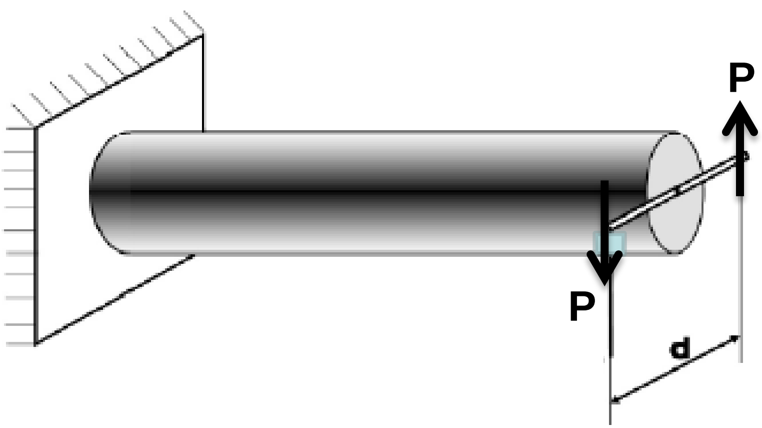


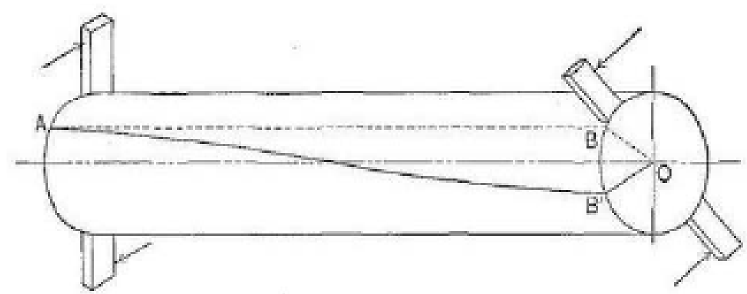
Basic Theory of Torsion

1. *We will derive the theory of torsion of circular shafts.*
2. *An example of torsion loading is shown here. In this example we load the shaft by two equal and opposite forces acting on a bar perpendicular to the shaft axis.*
3. *The moment generated by these forces is called a* ***twisting moment*** *. The magnitude of the moment due to this couple is given by*

$$T = P * d$$

*where, P is the applied force and d is the distance between the lines of action of the forces. This twisting moment is also called the '****Torque****'.*



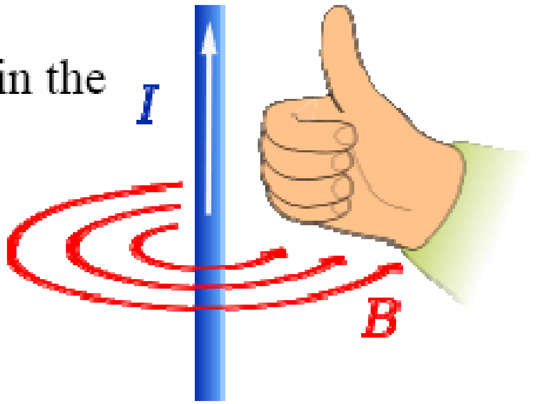


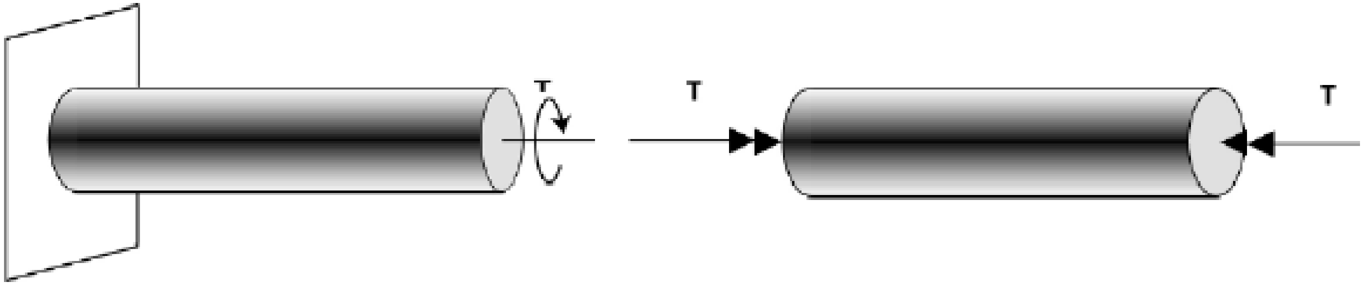
Alternate representations of torque

Two alternate ways of depicting torque are shown here.

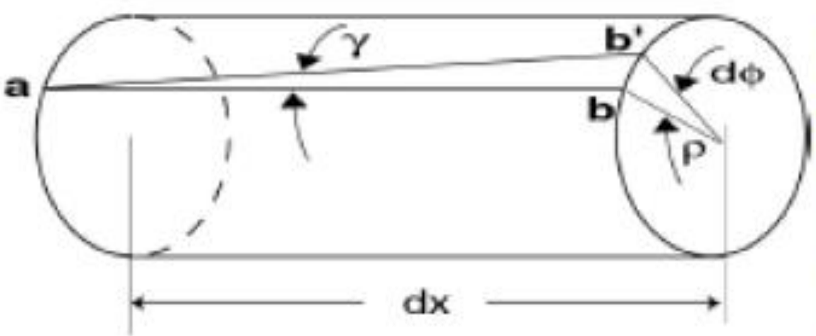
1. In the left-hand figure the torque is shown as a loop with an arrow depicting its direction.
2. In the right-hand figure the torque is shown as a vector moment. The direction of the moment is parallel to the shaft. The sign of the moment, can be understood using the right hand rule.

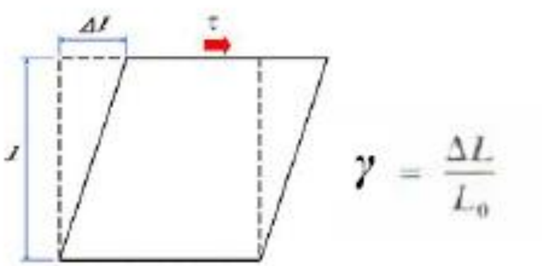
The ***right hand rule*** is that "if you rotate your right hand in the direction of the applied torque, then your thumb points in the direction of the vector indicated by two arrowheads in the right-hand figure.





Deformation due to torsion





Note: Relations here are based solely on geometry and so they are valid for circular shaft of any material, linear or non-linear, elastic or non-elastic.

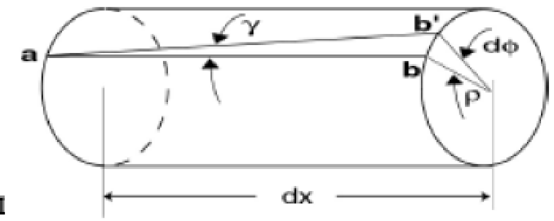
- ρ is the radial distance to any point.
- ϕ is the angle of twist in radians
- γ is the shear strain.
- The horizontal line ab moves to ab'
- The shear strain is $\gamma = \dfrac{bb'}{ab}$
- From geometry $\gamma dx = \rho d\phi$
- So the strain $\gamma = \rho \dfrac{d\phi}{dx}$

- Hence twist and strain are related as

$$\gamma = \frac{\rho\phi}{L}$$

3

Deformation due to torsion



The theory of torsion of circular shafts.

Look at a small section of length dx of a circular shaft under torsion. During twisting, one end of the shaft will rotate about the longitudinal axis with respect to the other end. The magnitude of this rotation is measured in terms of the angle (in radians) by which one end rotates relative to the other. This is called the *'Angle of Twist'.*

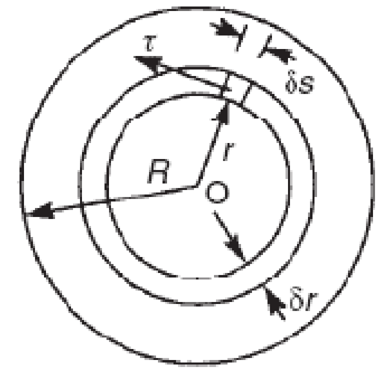
It can be seen that the line AB, which was initially horizontal, rotates through an angle γ, and moves to the line AB'. Here d φ is the "*angle of twist*".

The "*shear strain*", gamma is the angle between AB and AB'. It is found by the distance BB' divided by the distance AB. Using geometry, the arc length: $\gamma = \rho\, d\phi / dx$.

Let's assume that we are dealing with a shaft of uniform cross section and materials, thus the total twist, φ *over a length L is simply* $\gamma = \rho\, \phi / L$. This is the relation of shear strain γ to twist angle (φ), radial distance (ρ), and shaft length (L).

All the relations here, are based solely on the geometry of the circular shaft. Hence they are valid for any type of material. This is not so in what follows, the calculation of stresses based on linear elastic material behavior.

Stresses in Torsion



For a linear elastic material, using Hooke's law, we can write the shear stress using Hooke's law for a linear elastic material as

$$\tau = G * \gamma$$

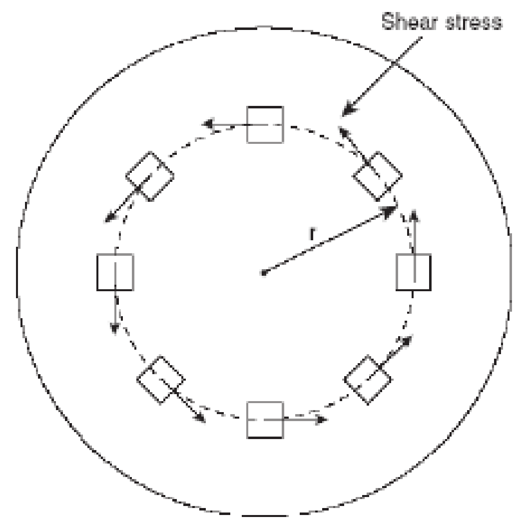
where, G is the Shear Modulus.

The shear strain on a small area of material situated at a distance ρ from the center, the shear strain is

$$\gamma = \rho \phi / L$$

and the shear stress is

$$\tau = G \rho \phi / L$$



*The torque, T, is calculated by integrating over the cross section the product of shear stress, τ, and the distance, ρ, from the center of the shaft. The torque, T, is found by integrating the shear stress along any ring "donut" of radius ρ * distance over the cross section, S, of the shaft*

$$T = \int_S \tau \rho \, dA$$

Relation of torque & angle of twist

Using stress from previous relations and substituting the stress from previous expressions, we find that torque is the integral of shear stresses over the cross section of the shaft.

$$T = \int_S \tau \rho \, dA$$

$$T = \int_S G\frac{\phi}{L}\rho^2 dA = G\frac{\phi}{L}\int_S \rho^2 dA = G\frac{\phi}{L}J$$

where J is the polar moment of inertia "will discuss polar moment of inertia, J, on the next slide".

Using the above we find the relation between the angle of angle of twist and the torque as:

$$\phi = \frac{TL}{GJ}$$

And the we can write the shear stress as: $\tau = G\frac{\rho\phi}{L} = G\frac{\rho}{L}\frac{TL}{JG} = \frac{T\rho}{J}$

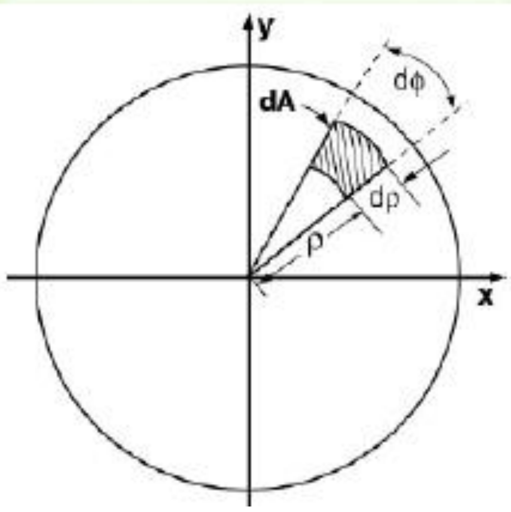
The theory of torsion compatibility may be rearranged as follows; $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$

Polar moment of Inertia

Definition: The Polar Moment of Inertia is defined as the integral

$$J = \int_S \rho^2 dA$$

If 'O' is the centroid of the area, then ρ Is the distance from the point 'O' to the element of area dA.



*Solid circular cross section:

$$J = \frac{\pi r^4}{2} \qquad \{r = radius\}$$

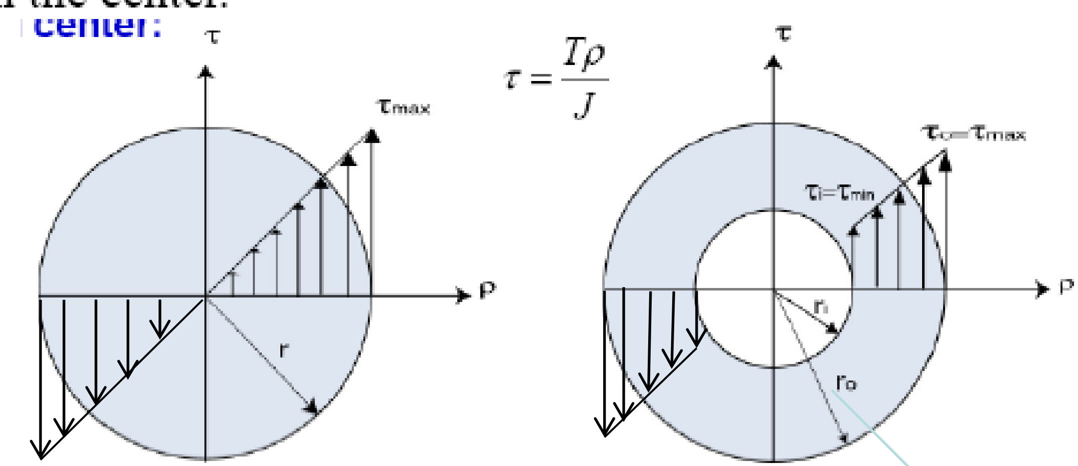
*Hollow Circular Cross Section:

$$J = \frac{\pi (r_o^4 - r_i^4)}{2}$$

where, $\begin{Bmatrix} r_o = & outer & radius \\ r_i = & inner & radius \end{Bmatrix}$

Torsion stress distribution in a circular cross section

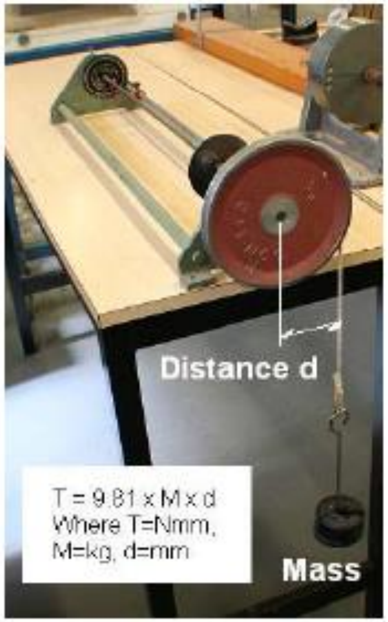
In a circular shaft shear stress varies linearly from center:
The shear stress distribution on a circular cross section under torsion loading. The shear stresses are directly proportional to the distance from the center. For a circular shaft, the shear stress would be maximum for an element which is farthest from the center.



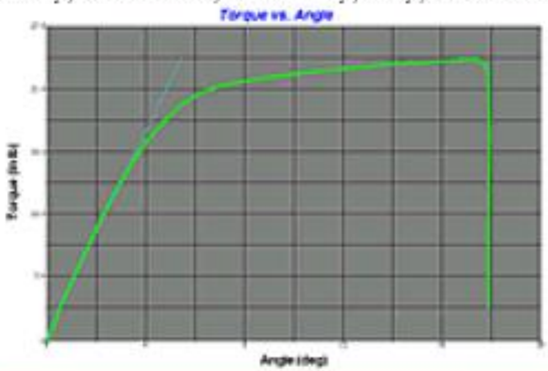
The shear stress is zero at the center while it is the maximum at the surface. For a hollow shaft the shear stress is minimum for on the inside surface and maximum on the outer surface.

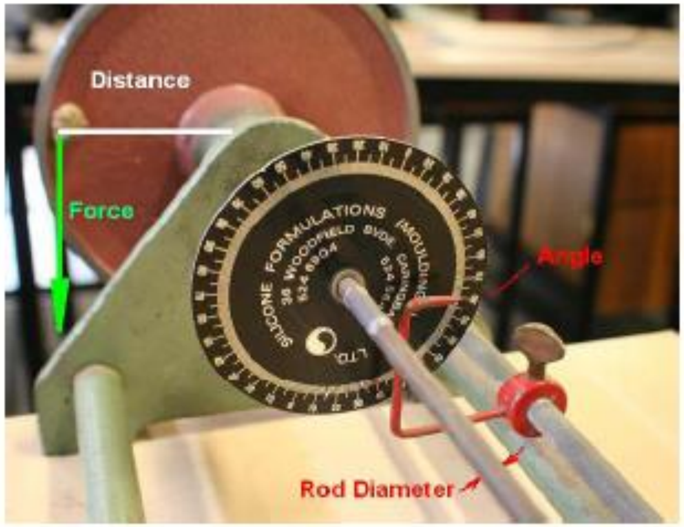
Note: The shear stress is maximum for the outermost element where the radii is maximum.

Performing of a torsion test



1. Torsion test is carried out by applying a twisting moment and measuring the corresponding twisting angle.
2. The specimen used is a solid pipe of diameter D and length L fixed from one end and attached to a pulley from the other end.
3. A mass "M" is hanged at a distance "d = D/2" producing a twisting moment "T = Mg * d"
4. The twisting angle θ is measured using a proctor attached to the specimen as shown.
5. The twisting moment, twisting angle relation is drawn.





Mode of Failure

Figure 16.6 Failure in torsion of a circular bar of brittle cast iron, showing a tendency to tensile fracture across a helix on the surface of the specimen.

1- Brittle Metals

The torsional failure of ductile materials occurs when the shearing stresses attain the yield stress of the material. The greatest shearing stresses in a circular shaft occur in a cross-section and along the length of the shaft. A circular bar of a ductile material usually fails by breaking off over a normal cross-section, as shown in Figure 16.7.

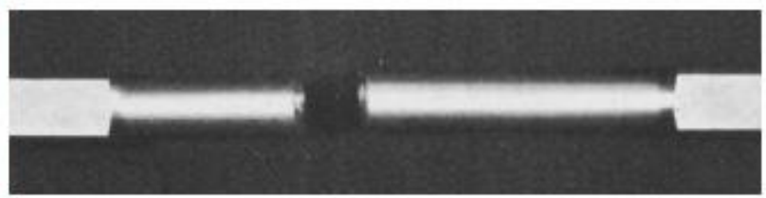


Figure 16.7 Failure of torsion of a circular bar of ductile cast iron, showing a shearing failure over a normal cross-section of the bar.

2- Ductile Metals

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Watching a torsion testing practice.

1

2

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The mechanical properties after torsion test.

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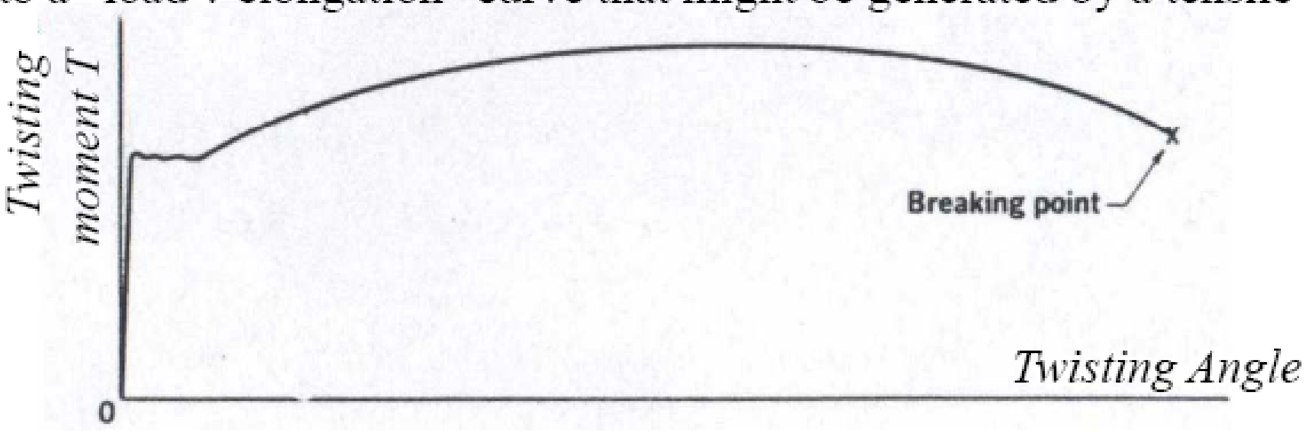
Why do we perform a torsion test?

Many products and components are subjected to torsion forces during their operation. Products such as switches, fasteners, and ***automotive steering columns*** are just a few devices subject to such torsion stresses.
By testing these products in torsion, manufacturers are able to simulate real life service conditions, check product quality, verify designs, and ensure proper manufacturing techniques.

The mechanical properties after torsion test.

- A torsion test can be conducted on most materials to determine the torsion properties of the material. These properties include but are not limited to:
- Modulus of elasticity in shear
- Yield shear strength
- Modulus of rupture in shear
- Modulus of resilience in shear
- Modulus of toughness in shear
- Ductility
- While they are not the same, they are analogous to properties that can be determined during a torsion test. In fact, the "torque versus angle" diagram looks very similar to a “load v elongation" curve that might be generated by a tensile test.



The mechanical properties after torsion test.

Proportional Limit, Yield Shear Strength

Proportional limit is the maximum shear stress where shear stress is proportional to the twisting angle.

Proportional limit torsion moment is defined as the end of the straight line. Thus, if the proportional limit torsion moment is M_{PL} and the original polar moment of inertia = J, then:

Proportional limit torsion moment $= \boldsymbol{T_{PL}}$

Proportional Limit shear stress $= \tau_{PL} = \boldsymbol{T_{PL} * R_{max} / J}$

R_{max} $= D_o/2$ for circular cross sections.

J $= \pi(D^4_o - D^4_i)/32$ for hollow circular sections.

$= \pi * D^4_o/32$ for circular cross sections.

Proportional Limit shear stress $= \tau_{PL} = 16 * \boldsymbol{T_{PL} / \pi * D_o^3}$

The mechanical properties after torsion test.

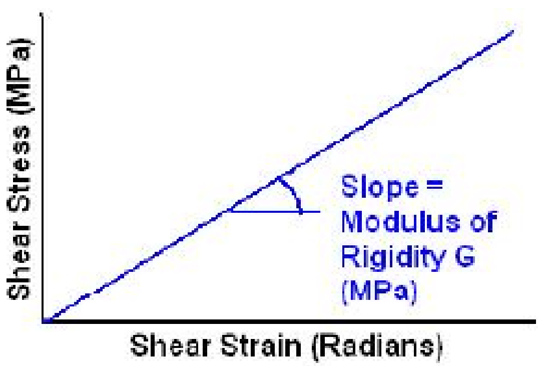
Shear Modulus, Modulus of Rigidity

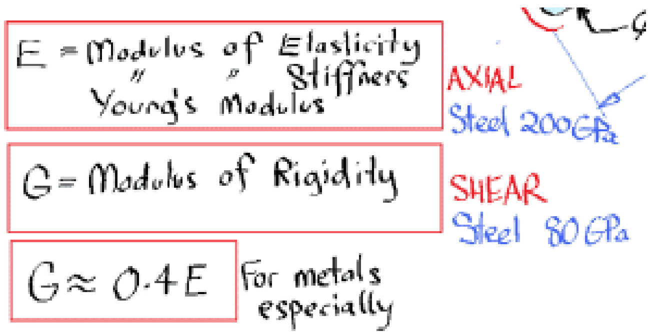
The torsion moment – twisting angle “load-deformation” diagram for most engineering materials exhibit a linear relationship between the applied twisting moment and twisting angle within the elastic region. Consequently, an increase in stress causes a proportionate increase in strain.

The relation between *the shear modulus*, applied twisting moment, measured twisting angle and the specimen property is given as follows;

$$\frac{G\,\theta}{L} = \frac{T_{PL}}{J} = \frac{\tau_R}{R} = \frac{\tau_r}{r}$$

$$G = \frac{T_{PL}\,L}{\theta_{PL} * J}$$





The mechanical properties after torsion test.

Modulus of rupture in Shear

Maximum Shear Strength is the stress of the extreme fiber of a specimen at its failure in the torsion Test.

There is no close-form solution for the shear stresses beyond elastic limit. But there are *empirical* formulae for the shear stress.

Thus, if the modulus of rupture in shear is calculated using the maximum applied torsion moment is T_{max} and the original polar moment of inertia = J, then:

Maximum applied torsion moment $= \boldsymbol{T_{max}}$

Modulus of rupture in shear (brittle metals) $= \tau_{R\,max} = 7/8 * \boldsymbol{T_{max}} * \boldsymbol{R_{max}} / \boldsymbol{J}$

Modulus of rupture in shear (ductile metals) $= \tau_{R\,max} = 3/4 * \boldsymbol{T_{max}} * \boldsymbol{R_{max}} / \boldsymbol{J}$

R_{max} = $D_o/2$ for circular cross sections.

J = $\pi(D^4_o - D^4_i)/32$ for hollow circular sections.

= $\pi * D^4_o/32$ for circular cross sections.

Modulus of rupture in shear (brittle metals) $= \tau_{R\,max} = 14 * \boldsymbol{T_{PL}} / \boldsymbol{\pi} * \boldsymbol{D_o^3}$

Modulus of rupture in shear (ductile metals) $= \tau_{R\,max} = 12 * \boldsymbol{T_{PL}} / \boldsymbol{\pi} * \boldsymbol{D_o^3}$

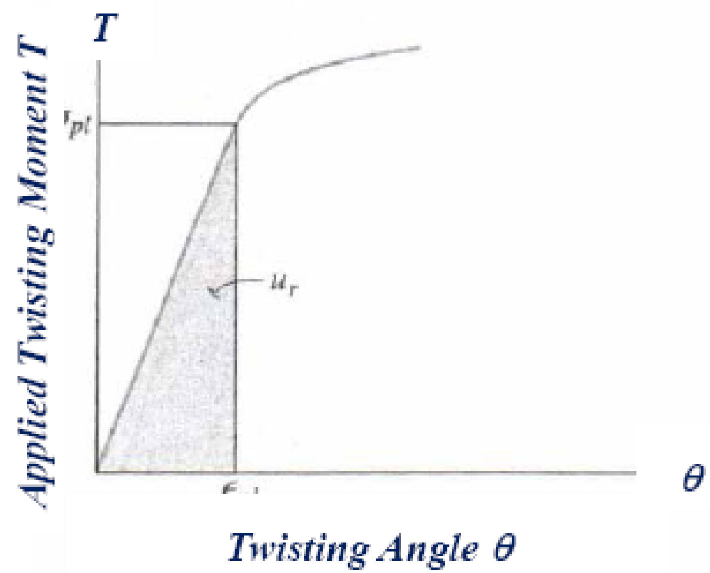
The mechanical properties after torsion test.

Modulus of Resilience

A material's resilience represents the ability of the material to absorb energy without any permanent damage to the material. In particular, when the load a reaches the proportional limit, the strain-energy density, is calculated by and is referred to as the *modulus of resilience Ur.* Mathematically it is the area under the straight line "elastic region" of the load-deformation curve per unit volume.

$$U_r = \frac{T_{PL} * \theta_{PL}}{2 * A * L}$$

θ_{PL} is in radians NOT degrees.



(a)

The mechanical properties after torsion test.

Modulus of Resilience

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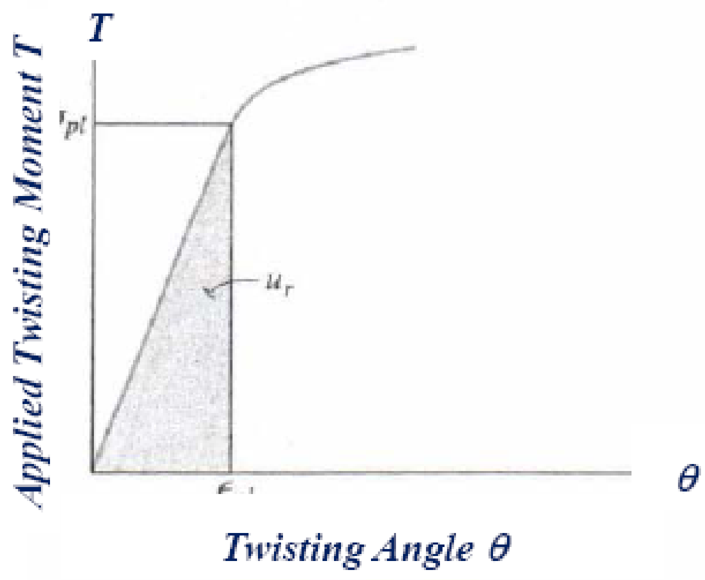

(a)

The mechanical properties after torsion test.

Ductility

Is defined as the extent to which a material can sustain plastic deformation without rupture. Maximum twisting angle is a common indices of ductility measured in radians.

$$Ductility = \theta_{max} \ \ (in\ radians).$$